7. $\begin{cases} 2x - y = 5 \\ 5x + 2y = 8 \end{cases}$

Substituting the values of the variables:

$$\begin{cases} 2(2) - (-1) = 4 + 1 = 5 \\ 5(2) + 2(-1) = 10 - 2 = 8 \end{cases}$$

Each equation is satisfied, so $x = 2,\ y = -1$ is a solution of the system of equations.

17. $\begin{cases} x + y = 8 \\ x - y = 4 \end{cases}$

Solve the first equation for y, substitute into the second equation and solve:

$$\begin{cases} y = 8 - x \\ x - y = 4 \end{cases}$$

$$\begin{aligned} x - (8 - x) &= 4 \\ x - 8 + x &= 4 \\ 2x &= 12 \\ x &= 6 \end{aligned}$$

Since $x = 6,\ y = 8 - 6 = 2$. The solution of the system is $x = 6,\ y = 2$.

19. $\begin{cases} 5x - y = 13 \\ 2x + 3y = 12 \end{cases}$

Multiply each side of the first equation by 3 and add the equations to eliminate y:

$$\begin{cases} 15x - 3y = 39 \\ 2x + 3y = 12 \end{cases}$$

$$\begin{aligned} 17x &= 51 \\ x &= 3 \end{aligned}$$

Substitute and solve for y:

$$\begin{aligned} 5(3) - y &= 13 \\ 15 - y &= 13 \\ -y &= -2 \\ y &= 2 \end{aligned}$$

The solution of the system is $x = 3,\ y = 2$.

$x = 3$
$y = 2$ $\Longleftarrow$

6.2 Matrix Methods

(7) Is $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ a sol'n?

$$\begin{bmatrix} 2 & -1 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 4+1 \\ 10-2 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix} \checkmark$$

Yes.

(17) $\left[\begin{array}{cc|c} 1 & 1 & 8 \\ 1 & -1 & 4 \end{array}\right] \begin{array}{l} R1 \\ -R1+R2 \end{array} \left[\begin{array}{cc|c} 1 & 1 & 8 \\ 0 & -2 & -4 \end{array}\right]$

$\begin{array}{l} R1 \\ -\frac{1}{2}R2 \end{array} \left[\begin{array}{cc|c} 1 & 1 & 8 \\ 0 & 1 & 2 \end{array}\right] \begin{array}{l} -R2+R1 \\ R2 \end{array} \left[\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 1 & 2 \end{array}\right]$

$\Longrightarrow x = 6,\ y = 2$ OR $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$

(19) $\left[\begin{array}{cc|c} 5 & -1 & 13 \\ 2 & 3 & 12 \end{array}\right]$

$\begin{array}{l} \frac{1}{5}R1 \\ R2 \end{array} \left[\begin{array}{cc|c} 1 & -\frac{1}{5} & \frac{13}{5} \\ 2 & 3 & 12 \end{array}\right]$

$\begin{array}{l} R1 \\ -2R1+R2 \end{array} \left[\begin{array}{cc|c} 1 & -\frac{1}{5} & \frac{13}{5} \\ 0 & \frac{17}{5} & \frac{34}{5} \end{array}\right]$

SCRATCH: $(-2)(-\frac{1}{5}) + 3 = \frac{2}{5} + 3$
$= \frac{2}{5} + \frac{15}{5} = \frac{17}{5}$
$(-2)(\frac{13}{5}) + 12 = \frac{-26}{5} + \frac{60}{5} = \frac{34}{5}$

$\begin{array}{l} R1 \\ \frac{5}{17}R2 \end{array} \left[\begin{array}{cc|c} 1 & -\frac{1}{5} & \frac{13}{5} \\ 0 & 1 & 2 \end{array}\right]$

$\begin{array}{l} \frac{1}{5}R2+R1 \\ R2 \end{array} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \end{array}\right]$

SCRATCH: $(\frac{1}{5})(2) + \frac{13}{5} = \frac{2}{5} + \frac{13}{5} = \frac{15}{5} = 3$

29. $\begin{cases} x+2y=4 \\ 2x+4y=8 \end{cases}$

Solve the first equation for x, substitute into the second equation and solve:

$\begin{cases} x=4-2y \\ 2x+4y=8 \end{cases}$

$$2(4-2y)+4y=8$$
$$8-4y+4y=8$$
$$0y=0$$

These equations are dependent. Any real number is a solution for y. The solution of the system is $x=4-2y$, where y is any real number.

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$$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 2 & 4 & 8 \end{array}\right]$$

$$\begin{array}{r} R1 \\ -2R1+R2 \end{array}\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 0 & 0 \end{array}\right] \; !$$

So $x+2y=4$ is IT!

$\Rightarrow x=-2y+4 \Rightarrow$

Sol'n Set is $\{(-2y+4,\, y) \mid y \in \mathbb{R}\}$

35. $\begin{cases} \frac{1}{2}x+\frac{1}{3}y= \ \ 3 \\ \frac{1}{4}x-\frac{2}{3}y=-1 \end{cases}$

Multiply each side of the first equation by –6 and each side of the second equation by 12, then add to eliminate x:

$\begin{cases} -3x-2y=-18 \\ \ \ 3x-8y=-12 \end{cases}$

$$-10y=-30$$
$$y=3$$

Substitute and solve for x:

$$\frac{1}{2}x+\frac{1}{3}(3)=3$$
$$\frac{1}{2}x+1=3$$
$$\frac{1}{2}x=2$$
$$x=4$$

The solution of the system is $x=4$, $y=3$

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$$\left[\begin{array}{cc|c} \frac{1}{2} & \frac{1}{3} & 3 \\ \frac{1}{4} & -\frac{2}{3} & -1 \end{array}\right] \begin{array}{l} 2R1 \\ R2 \end{array}\left[\begin{array}{cc|c} 1 & \frac{2}{3} & 6 \\ \frac{1}{4} & -\frac{2}{3} & -1 \end{array}\right]$$

$$\begin{array}{r} R1 \\ -\frac{1}{4}R1+R2 \end{array}\left[\begin{array}{cc|c} 1 & \frac{2}{3} & 6 \\ 0 & -\frac{5}{6} & -\frac{5}{2} \end{array}\right]$$

$$\left(-\frac{1}{4}\right)\left(\frac{2}{3}\right) + -\frac{2}{3} = -\frac{1}{6} - \frac{2}{3} = -\frac{1}{6} - \frac{4}{6} = -\frac{5}{6}$$

$$\left(-\frac{1}{4}\right)(6) + -1 = -\frac{3}{2} - 1 = -\frac{3}{2} - \frac{2}{2} = -\frac{5}{2}$$

$$\begin{array}{r} R1 \\ -\frac{6}{5}R2 \end{array}\left[\begin{array}{cc|c} 1 & \frac{2}{3} & 6 \\ 0 & 1 & 3 \end{array}\right]$$

$$\begin{array}{r} -\frac{2}{3}R2+R1 \\ R2 \end{array}\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 3 \end{array}\right]$$

$(x,y) = (4,3)$

OR

$(x,y) \in \{(4,3)\}$ = Sol'n set.

Scratch:

$$\left(-\frac{2}{3}\right)(3) + 6 = -2 + 6 = 4$$

6.2 MATRIX METHODS

41. $\begin{cases} x - y = 6 \\ 2x - 3z = 16 \\ 2y + z = 4 \end{cases}$

Multiply each side of the first equation by -2 and add to the second equation to eliminate x:

$$\begin{array}{r} -2x + 2y \quad = -12 \\ \underline{2x \quad -3z = \ \ 16} \\ 2y - 3z = \ \ 4 \end{array}$$

Multiply each side of the result by -1 and add to the original third equation to eliminate y:

$$\begin{array}{r} -2y + 3z = -4 \\ \underline{2y + \ z = \ \ 4} \\ 4z = \ \ 0 \\ z = 0 \end{array}$$

NEW SYSTEM:

$$\begin{array}{l} x - y = 6 \\ 2y - 3z = 4 \\ 2y + z = 4 \end{array}$$

Substituting and solving for the other variables:

$$\begin{array}{ll} 2y + 0 = 4 & 2x - 3(0) = 16 \\ 2y = 4 & 2x = 16 \\ y = 2 & x = 8 \end{array}$$

The solution is $x = 8,\ y = 2,\ z = 0$.

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$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 6 \\ 2 & 0 & -3 & 16 \\ 0 & 2 & 1 & 4 \end{array}\right]$$

$$\begin{array}{r} R1 \\ -2R1 + R2 \\ R3 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 6 \\ 0 & 2 & -3 & 4 \\ 0 & 2 & 1 & 4 \end{array}\right]$$

$$\begin{array}{r} R1 \\ \frac{1}{2}R2 \\ -R2 + R3 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 6 \\ 0 & 1 & -\frac{3}{2} & 2 \\ 0 & 0 & 4 & 0 \end{array}\right]$$

$$\begin{array}{r} R2 + R1 \\ R2 \\ \frac{1}{4}R3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{3}{2} & 8 \\ 0 & 1 & -\frac{3}{2} & 2 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{array}{r} \frac{3}{2}R3 + R1 \\ \frac{3}{2}R3 + R2 \\ R3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$(x, y, z) \in \{(8, 2, 0)\}$

45. $\begin{cases} x - y - z = 1 \\ 2x + 3y + z = 2 \\ 3x + 2y \quad = 0 \end{cases}$

Add the first and second equations to eliminate z:

$$\begin{array}{r} x - \ y - z = 1 \\ \underline{2x + 3y + z = 2} \\ 3x + 2y \quad = 3 \end{array}$$

Multiply each side of the result by -1 and add to the original third equation to eliminate y:

$$\begin{array}{r} -3x - 2y = -3 \\ \underline{3x + 2y = \ \ 0} \\ 0 = -3 \end{array}$$

This result has no solution, so the system is inconsistent.

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$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 2 & 3 & 1 & 2 \\ 3 & 2 & 0 & 0 \end{array}\right]$$

$$\begin{array}{r} R1 \\ -2R1 + R2 \\ -3R1 + R3 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 0 & 5 & 3 & 0 \\ 0 & 5 & 3 & -3 \end{array}\right]$$

$$\begin{array}{r} R1 \\ R2 \\ -R2 + R3 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 0 & 5 & 3 & 0 \\ 0 & 0 & 0 & -3 \end{array}\right]$$

$\Longrightarrow 0 = -3$!? FALSE! NO SOL'N!

6.2 Assignment – Do 6.1 with matrices!

47. $\begin{cases} x-y-z=1 \\ -x+2y-3z=-4 \\ 3x-2y-7z=0 \end{cases}$

Add the first and second equations to eliminate x: multiply the first equation by -3 and add to the third equation to eliminate x:

$$\begin{aligned} x-y-z&=1 \\ -x+2y-3z&=-4 \\ \hline y-4z&=-3 \end{aligned}$$

$$\begin{aligned} -3x+3y+3z&=-3 \\ 3x-2y-7z&=0 \\ \hline y-4z&=-3 \end{aligned}$$

Multiply each side of the first result by -1 and add to the second result to eliminate y:

$$\begin{aligned} -y+4z&=3 \\ y-4z&=-3 \\ \hline 0&=0 \end{aligned}$$

The system is dependent. If z is any real number, then $y=4z-3$.

Solving for x in terms of z in the first equation:

$$x-(4z-3)-z=1$$
$$x-4z+3-z=1$$
$$x-5z+3=1$$
$$x=5z-2$$

The solution is $x=5z-2$, $y=4z-3$, z is any real number.

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$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ -1 & 2 & -3 & -4 \\ 3 & -2 & -7 & 0 \end{array}\right]$$

$$\begin{array}{c} R1 \\ R1+R2 \\ -3R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 0 & 1 & -4 & -3 \\ 0 & 1 & -4 & -3 \end{array}\right]$$

$$\begin{array}{c} R1 \\ R2 \\ -R2+R1 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 0 & 1 & -4 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{c} R2+R1 \\ R2 \\ R3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & -5 & -2 \\ 0 & 1 & -4 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\Rightarrow \quad x-5z=-2$$
$$y-4z=-3$$
$$\Rightarrow \quad x=5z-2$$
$$y=4z-3$$

$\boxed{\Rightarrow (x,y,z) \text{ is in the set } \{(5z-2, 4z-3, z) \mid z\in\mathbb{R}\}}$

57. Cost of Fast Food Four large cheeseburgers and two chocolate shakes cost a total of \$7.90. Two shakes cost 15¢ more than one cheeseburger. What is the cost of a cheeseburger? What is the cost of a shake?

57. Let x = the cost of one cheeseburger and y = the cost of one shake. Then:
$4x + 2y = 790$ and $2y = x + 15$

Solve by substitution:

$$\begin{array}{ll} 4x + x + 15 = 790 & 2y = 155 + 15 \\ 5x = 775 & 2y = 170 \\ x = 155 & y = 85 \end{array}$$

A cheeseburger cost \$1.55 and a shake costs \$0.85.

Author isn't explicit about units. Clearly, he's using "cents," that is x is the number of pennies a cheeseburger costs! I use dollars:

x = cost of a cheeseburger (in \$)
y = cost of a shake (also in \$).

Break it down. Turn the following two sentences into equations:

1. Four large cheeseburgers and two chocolate shakes cost a total of \$7.90.

$4x + 2y = 7.90$

2. Two shakes cost 15¢ more than one cheeseburger.

$2y = x + .15$

These two equations give

$$\begin{array}{l} 4x + 2y = 7.9 \\ -x + 2y = .15 \end{array} \longrightarrow \left[\begin{array}{cc|c} 4 & 2 & 7.9 \\ -1 & 2 & .15 \end{array}\right]$$

$$R1 \longleftrightarrow R2 \left[\begin{array}{cc|c} -1 & 2 & .15 \\ 4 & 2 & 7.9 \end{array}\right] \begin{array}{c} -R1 \\ 4R1+R2 \end{array} \left[\begin{array}{cc|c} 1 & -2 & -.15 \\ 0 & 10 & 8.5 \end{array}\right]$$

$$\begin{array}{c} R1 \\ \frac{1}{10}R2 \end{array} \left[\begin{array}{cc|c} 1 & -2 & -.15 \\ 0 & 1 & .85 \end{array}\right] \begin{array}{c} 2R2+R1 \\ R2 \end{array} \left[\begin{array}{cc|c} 1 & 0 & 1.55 \\ 0 & 1 & .85 \end{array}\right]$$

$\Longrightarrow (x,y) = (1.55, .85)$, i.e.,
Cheeseburger costs \$1.55 & Shake costs \$.85

69. Curve Fitting Find real numbers a, b, and c so that the graph of the function $y = ax^2 + bx + c$ contains the points $(-1, 4)$, $(2, 3)$, and $(0, 1)$.

69. $y = ax^2 + bx + c$

At $(-1, 4)$ the equation becomes:

$4 = a(-1)^2 + b(-1) + c$

$4 = a - b + c$

At $(2, 3)$ the equation becomes:

$3 = a(2)^2 + b(2) + c$

$3 = 4a + 2b + c$

At $(0, 1)$ the equation becomes:

$1 = a(0)^2 + b(0) + c$

$1 = c$

The system of equations is:

$$\begin{cases} a - b + c = 4 \\ 4a + 2b + c = 3 \\ c = 1 \end{cases}$$

Substitute $c = 1$ into the first and second equations and simplify:

$a - b + 1 = 4 \qquad 4a + 2b + 1 = 3$

$a - b = 3 \qquad 4a + 2b = 2$

$a = b + 3$

Solve the first result for a, substitute into the second result and solve:

$4(b+3) + 2b = 2$

$4b + 12 + 2b = 2$

$6b = -10$

$b = -\frac{5}{3}$

$a = -\frac{5}{3} + 3 = \frac{4}{3}$

The solution is $a = \frac{4}{3}$, $b = -\frac{5}{3}$, $c = 1$. The equation is $y = \frac{4}{3}x^2 - \frac{5}{3}x + 1$.

So, we plug in each (x,y)-pair into the equation. Hope it's clear what's being done. My focus is the matrix algebra:

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 4 & 2 & 1 & 3 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\begin{array}{r} R1 \\ -4R1+R2 \\ R3 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 0 & 6 & -3 & -13 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\begin{array}{r} R1 \\ \frac{1}{6}R2 \\ R3 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 0 & 1 & -\frac{1}{2} & -\frac{13}{6} \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\begin{array}{r} R2+R1 \\ R2 \\ R3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & \frac{11}{6} \\ 0 & 1 & -\frac{1}{2} & -\frac{13}{6} \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$-\frac{13}{6} + 4 = -\frac{13}{6} + \frac{24}{6} = \frac{11}{6}$

$$\begin{array}{r} -\frac{1}{2}R3+R1 \\ \frac{1}{2}R3+R2 \\ R3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{4}{3} \\ 0 & 1 & 0 & -\frac{5}{3} \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$\left(-\frac{1}{2}\right)(1) + \frac{11}{6} = -\frac{1}{2} + \frac{11}{6} = -\frac{3}{6} + \frac{11}{6} = \frac{8}{6} = \frac{4}{3}$

$\left(\frac{1}{2}\right)(1) + -\frac{13}{6} = \frac{1}{2} - \frac{13}{6} = \frac{3}{6} - \frac{13}{6} = -\frac{10}{6} = -\frac{5}{3}$

So, $a = \frac{4}{3}$, $b = -\frac{5}{3}$, $c = 1$ &
$y = \frac{4}{3}x^2 - \frac{5}{3}x + 1$ is desired quadratic function.

75. Theater Revenues A Broadway theater has 500 seats, divided into orchestra, main, and balcony seating. Orchestra seats sell for \$50, main seats for \$35, and balcony seats for \$25. If all the seats are sold, the gross revenue to the theater is \$17,100. If all the main and balcony seats are sold, but only half the orchestra seats are sold, the gross revenue is \$14,600. How many are there of each kind of seat?

I would prefer that the author not eliminate "z" before eliminating "x". It's a good, legal trick, but too many students lose their way and fail to finish when they're not "systematic," as I'm trying to be.

75. Let x = the number of orchestra seats.
Let y = the number of main seats.
Let z = the number of balcony seats.
Since the total number of seats is 500,
$x+y+z=500$.

Since the total revenue is \$17,100 if all seats are sold, $50x+35y+25z=17,100$.

If only half of the orchestra seats are sold, the revenue is \$14,600.

So, $50\left(\frac{1}{2}x\right)+35y+25z=14,600$.

Thus, we have the following system:

$$\begin{cases} x+y+z=500 \\ 50x+35y+25z=17,100 \\ 25x+35y+25z=14,600 \end{cases}$$

Multiply each side of the first equation by –25 and add to the second equation to eliminate z; multiply each side of the third equation by –1 and add to the second equation to eliminate z:

$$\begin{array}{r} -25x-25y-25z=-12,500 \\ 50x+35y+25z=17,100 \\ \hline 25x+10y=4600 \end{array}$$

$$\begin{array}{r} 50x+35y+25z=17,100 \\ -25x-35y-25z=-14,600 \\ \hline 25x=2500 \\ x=100 \end{array}$$

Substituting and solving for the other variables:

$$\begin{aligned} 25(100)+10y&=4600 \\ 2500+10y&=4600 \\ 10y&=2100 \\ y&=210 \end{aligned} \qquad \begin{aligned} 100+210+z&=500 \\ 310+z&=500 \\ z&=190 \end{aligned}$$

There are 100 orchestra seats, 210 main seats, and 190 balcony seats.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 500 \\ 50 & 35 & 25 & 17100 \\ 25 & 35 & 25 & 14600 \end{array}\right]$$

$$\begin{array}{r} R1 \\ -50R1+R2 \\ -25R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 500 \\ 0 & -15 & -25 & -7900 \\ 0 & 10 & 0 & 2100 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 500 \\ 0 & 1 & 0 & 210 \\ 0 & -15 & -25 & -7900 \end{array}\right]$$

$$\begin{array}{r} -R2+R1 \\ R2 \\ 15R2+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 290 \\ 0 & 1 & 0 & 210 \\ 0 & 0 & -25 & -4750 \end{array}\right]$$

$$\begin{array}{r} R1 \\ R2 \\ -\frac{1}{25}R3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 290 \\ 0 & 1 & 0 & 210 \\ 0 & 0 & 1 & 190 \end{array}\right]$$

$$\begin{array}{r} -R3+R1 \\ R2 \\ R3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 100 \\ 0 & 1 & 0 & 210 \\ 0 & 0 & 1 & 190 \end{array}\right]$$

$$\Rightarrow \boxed{(x,y,z)=(100,210,190)}$$